A NUMBER PLAY IN THREE ACTS

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A NUMBER PLAY IN THREE ACTS[1]

ACT ONE.

The curtain rises upon a home scene. Three children, a boy and two girls are working their problems at a table near the center of the stage. The mother is sewing near by and the grandfather is reading. The children are Jane, Evelyn, and Willis.

Jane—I can't get this problem.

Evelyn—Neither can I. The work is awfully hard these days.

Willis—Oh, girls don't know much about mathematics anyway.

Evelyn—If you're so smart let us see how much you have done.

Willis—Oh, I do all my problems in my head.

Jane—Work this example then.

Willis—(After a long struggle) You girls make me tired. Work your own problems.

Jane—He can't do it.

Evelyn—I knew he couldn't.

Mother—Here, here, children, stop your quarreling, and get to work; it will be bed time in ten minutes.

Evelyn—But mother, we will never get our work done in ten minutes. We have an awful lot to do.

Grandfather—Well, hurry and finish, your work and I will tell you a story.

Willis—Great.

Evelyn—Oh, that will be fine.

Jane—I'll hurry. (Children work steadily for a short time.)

Evelyn—I've finished.

Jane—Just a minute and I'll be through.

Willis—Oh, I'll stop.

Grandfather—Well are you ready for your story?

Children—Yes we're ready.

Grandfather—Well, while you have been fretting over your lessons, I've been thinking. About how long do you think it took the world to learn to count?

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[1] Dramatized by an eighth grade mathematics class in the Ben Blewett Junior High School, St. Louis, Mo. Based upon "Number Stories of Long ago," by David Eugene Smith. Contributed by T. Schlierholz.

Willis—Oh about ten years.

Evelyn—Oh more than that.

Jane—I guess it took a hundred years.

Grandfather—Well you are all wrong. It took a great many hundred years. It is so long ago that not even the wisest men of China can tell the year or even the century in which little Ching, the king's eldest son, played at the foot of Mount Yu, and told the soldiers who guarded him, what a lot of turtles he had. He really had only three, but he did not know a name for three, so he just said, "One, two, a lot."

Willis—Did they never need to count any more?

Grandfather—No. We use our numbers principally in trading. The people in those days did not need numbers further than that because they did not trade. The kings had many slaves and they sent them out to kill animals and bring them back for food and clothing.

At the time that Ching was growing up in China, there was another little boy named Anem, who lived in the plains of Mesopotamia, in southern Asia. He helped to tend his father's flocks, and drove away the wolves that prowled around at night. One day, Bel, his father, called out to Anem. "There are a great many sheep out there; drive them back." There were only a few, but Anem could not count past three, so he said, "one, two, three," and then "many."

There was also another boy who lived on the banks of the Nile, in a house grander than any he or his father had ever seen. It really had only one room.

Jane—Only one room?

Grandfather—Yes, because that was before people had real houses. He felt very proud when he said, "What a lot of palm trees we have around our house." There were really only six, but neither he nor his father could count past four, so he would just say, "One, two, three, four," and then "a great many."

Evelyn—Well did they never learn to count past four?

Mother—Oh yes, but it took them many years to learn to count further. The Chinese boys learned to count by twos. "One, two, two and one, two twos, two twos and one, two twos and two, a lot."

And the Babylonian boys counted by threes; "One, two, three, three and one, three and two, two threes, two threes and one, two threes and two, three threes; three threes and one, three threes and two, many.''

The Egyptian boys counted by fives: ''One, two, three, four, five; five and one, five and two, five and three'' and so on, to ''five fives and four.''

Willis—Oh, he was the best of all; he got to twenty-five.

Grandfather—Yes, it took the world a long time to learn to count. Some people learned to count by twelves, and we still have twelve inches in a foot, and twelve things in a dozen.

Willis—And some people say twelve ounces in a pound.

Grandfather—Yes these things have staid with us. In warm countries near the equator, where the people wore no shoes they counted their toes as well as their fingers. They had names for all the numbers through nineteen, and then they would say, ''man finished.''

Jane—We, still say score and that is twenty.

Evelyn—And the French say four twenties and that is eighty.

Grandfather—Yes that is true. But the reason the whole world today counts by tens is that the Chings, and the Anams, and you and I, and all the children of the world have ten fingers. The early Romans used their fingers in counting. (Shows them how.) I. II. III. IIII. V. VI. VII. VIII. VIIII X. Later they used letters to represent these numbers.

Evelyn—This is all so interesting, do tell us some more.

Grandfather—No, not tonight, but tomorrow night I will tell you another story. And now we must away to Dreamland.—Curtain.

ACT TWO.

The rear wall of the stage is covered with charts showing how numbers were first represented by various peoples. There are also charts showing how people multiplied and divided. These charts are carefully explained by the characters as necessary. Blackboard, chalk, and pointer are at hand. Enter—Grandfather, Mother, and children. Ancients are hidden behind curtain drapery.

Jane—Why what a strange place!

Willis—I never saw anything so queer.

Evelyn—But where are we?

Grandfather—It seems to me that we are in the mysterious Land of Numbers. (Curtain rolls back, showing two groups Egypt, Arabia, Hebrew. Multiplication, and Division on one side; China, Greece, Rome, Fractions, and Decimals on the other side of the stage.)

Evelyn—Why this looks like an Egyptian. (Leads her to the center of the stage.)

Egypt—Yes I am Ancient Egypt. I was living four thousand years ago. Our people were the first to know much about mathematics. Our tax collectors had to know how to measure the land, because each time the Nile overflowed its banks, it changed the shape of each farmers land. We thought that we knew how to find the areas of the very shapes that bother you today, and strange to say our answers were nearly correct.

One of our greatest mathematicians was Ahmes. When he was a small boy he used to gaze at the Temple walls, and wish he could read what was written upon them. One day when Ahmes saw a priest reading the numbers, he asked him if he would teach him to read and write these numbers. The priest told him to get some papyrus and writing fluid.

Willis—What is papyrus?

Evelyn—Oh, I know what that is. It is a water plant that grows in Egypt. The Egyptians made a kind of paper out of the leaves by pressing them together.

Egypt—Fine. You know almost enough to have lived when Ahmes did. Ahmes learned to write his numbers this way. (Chart p. 16) This is the way he wrote 27,529 (Chart p. 20). On the temple walls were numbers like these giving the dates of the great wars and the kings' reigns.

When Ahmes grew to be a man, he became a priest and wrote the first book on mathematics. It is believed that he copied a book written still earlier. One of the examples in Ahmes' book was, "Heap its $1/7$, its whole, it makes 19." (Chart). You would say, "Add its $1/7$ to a number and the result will be 19." In algebra you write, $x + x/7 = 19$. Now when you make excavations in Egypt you find that mathematics was known by us much earlier than you had ever realized.

Jane—Why, this looks like a Chinaman.

China—Yes, I represent China. I am going to tell you how my forefathers wrote their numbers. It was many a long century ago, after the last of the Chings had learned to count by tens that there lived on the banks of the Yellow River a potter, whose plates and cups were known all over Shantung as the best to be found anywhere in that great province. One day Chang watched his father write with a brush on a palm leaf, some marks that showed how many cups he had on the top shelf of his shop.

Now Chang had learned to count the cups, but he could write ten only by making ten marks. Chang said to his father, "I want to learn to write the numbers as you do." Then his father took the brush, moistened it, rubbed it on a small cake of black paint, and painted these strange figures. (Chart p. 14) This is the way he would have written 789 (Chart p. 14) This stands for 7, this for 100, this for 8, this for 10, making 280, and this for 9 making 789.

Now while Chang was learning to write, another boy named Wu, was learning to add and subtract with these numbers. He too found it necessary to use something like the Calculi, which the Romans used; but instead of pebbles he used bamboo rods. He thought that he was doing something wonderful if he added two numbers in two minutes, for you must remember that he had only a little pile of sticks to use. It took a long time to lay out one number and it took still longer to lay out two numbers, and then find their sum (Chart p. 52). This is the way 1267 would have looked laid out in bamboo rods.

It was many more than 1,000 years after Wu learned to use bamboo rods that we adopted the Roman idea of having the calculi fastened to an abacus, and so we invented our reckoning board, or "swan-pan." This we use in the schools, banks, and shops throughout all China even to this day, and we can do our mathematics more rapidly in this way, than some of you can do yours with pencil and paper.

Hebrew—I represent the Ancient Hebrews. We did not have much mathematics. Little Daniel lived at the foot of the Mount of Olives. He was a Hebrew. He had to count because he helped his father sell figs and other fruits at the market. The prices were written on boards or pieces of parchment and placed above the fruits. They did not need numbers above ten or fifteen. The figures he learned were the first few letters of the Hebrew alphabet (Chart p. 28) just as the first numbers that Hippias learned were the first few letters of the Greek alphabet.

Greece—I am Ancient Greece. Egypt may have been the first to measure the land, but I had my great mathematicians also. Hippias learned to write his numbers on parchment. This is the way (Chart p. 26) he wrote 1, 5, 10, 100, 1000. If he wanted to write a number like 2,977 he had to use fifteen Greek letters. (Chart p. 26) Two thousand years ago another Hippias wrote on his parchment roll the Greek letters for numerals. (Chart p. 27) These little marks ' distinguish the letters from the numerals.

Thales lived 600 years before Christ. He introduced geometry into Greece. Thales was a merchant, and while he was travelling through Egypt, he became interested in mathematics. There were many queer stories told about Thales. One time when traveling through Egypt with a train of donkeys carrying salt, one of them slipped and fell into the stream. When he got up his load was lighter. The next time he came to a stream he did the same thing. To cure the donkey of this habit, Thales put a load of sponges upon the donkey's back. This time he again slipped into the water, but on getting up he found that his load was heavier. You may be sure the donkey did not try this trick again. Thales amazed the king of Egypt by measuring the height of the pyramids by measuring their shadows.

We all remember Pythagoras, because he showed us how to find the hypotenuse of a right triangle. He showed us that if we add the square of the base to the square of the altitude, we would have the square of the hypotenuse. Of course he did this by geometry. Pythagoras lived about 500 B. C. The Pythagareans had formed a secret group. If any member of the group discovered anything interesting he gave it to the credit of the whole group. You can learn something of group spirit from the Pythagoreans.

Euclid lived about 300 B. C. He wrote ten books called the Elements of Geometry. The geometry we study today is almost the same as Euclid gave it to us. In fact, today when an English boy wishes to say he is studying geometry, he says he is studying his Euclid. You must find out something about Archimedes also.

I want you all to know that Greece had a great woman mathematician also. Her name was Hypathia. She succeeded her father Theon, as a teacher. Well, Greece did a lot to help along the study of mathematics, but I must confess that we got our start from Egypt.

Rome—The Romans also did their share for mathematics. We did not make any great discoveries like the Egyptians and the Greeks did, but we knew how to use these discoveries and kept alive our interest in mathematics. Our warriors and merchants visited these countries and brought back this knowledge of mathematics. In time we became the greatest nation of the world and helped to spread this knowledge throughout Europe.

Titus who became one of our great men, was taught the numerals that were used in the shops. This is the old way of writing the numbers (Chart p. 28) and this is the later way. (Chart p. 29) If Titus had been asked to write a number larger than 1000, he would have been very much puzzled. This (Chart p. 29) is how he might have written 2,752,899. This is how he added 275 and 369. (Chart p. 47).

Titus had a friend named Caius, who attended a business school and learned to add and subtract, the only operations that were considered necessary. They did not know how to multiply or divide as you do today. Caius used to add and count by drawing lines on boards, letting them represent units, tens, and hundreds. (Chart p. 48) He would then place pebbles on the lines. Caius spoke in Latin, and his word for pebble was calculi, so instead of saying "Caius pebbled his answer," you would say "Caius calculated his answer."

Anyone who had any business to do would carry a small bag or box of calculi around with him. They used to carry boards with grooves cut into them, where they could place their pebbles. Thus we had our first adding machine.

Arabia—Don't you want to hear from Arabia?

All—Oh, yes, do tell us about your country.

Arabia—Gupta was one of our great men of the early days. The priests soon noticed that Gupta was brighter than the other boys of the village, so they taught him the numbers up to four, for that was as far as they themselves knew. (Chart p. 35). This is the way he wrote them. He learned to write with a sharp piece of wood on a palm leaf. They wrote their books in this way. (Shows a sample.)

When Gupta was a little older he learned to write another kind of numeral, and taught them to other people. (Charts p. 37) No one could write a number like 207 because the zero was not invented until long after Gupta died.

In France, nearly 1000 years ago, there lived a boy named Gebert. He was a good student, so the priests sent him to Spain with a nobleman whom they knew. Here he met some Arabs who knew the Hindu numerals. He learned these and later when he went to Rome he explained them to others.

Not much is known about the very early Arabian and Hindu mathematics. Some manuscripts were found written on birch bark. These show that they had found out that $\pi=3.1416$. One of our interesting men was Baskara. He wrote a book on mathematics, and called it Liliavati, after his daughter. This book was written in verse. Here is one of the problems;—"Lovely and dear Liliavali, whose eyes are like fauns, tell me, auspicious damsel, what are the numbers resulting from 135 multiplied by 12? Tell me what is the quotient of the same product divided by the same multiplier?

The word "algebra" came from the name of an Arabian book on mathematics. The Arabs studied and treasured the mathematics of Europe and Egypt. Later the people of Europe were so much interested in wars that learning was neglected. Many manuscripts were lost or destroyed. When the Moors entered Spain they established schools there. Many men from other countries studied in these schools. Thus the Arabs were able to give back to Europe much of her knowledge. These manuscripts were translated into Italian, and the Hindu-Arabic numerals were used in these translations. In this way they were introduced into Europe. We think we gave the world a most valuable gift when we gave her these numerals.

Evelyn—(Seeing Multiplication and Division) Why, who are these strang people?

Jane—(Discovering Fractions and Decimals.) And here are two others.

Multiplication—I am Multiplication. I had a very hard time to become known. People did not know who I was for a very long time. They could only add and subtract with Roman numerals. Some people used the calculus, but that was too difficult. I shall tell you about three boys who wanted to learn how to multiply. They were Leonardo, Cuthbert, and Johann.

Leonardo lived in Pisa. He learned arithmetic from a Moorish teacher who could multiply very rapidly. Later Leonardo wrote an arithmetic. This is how he multiplied. (Chart p. 66) First the multiplicand, then the multiplier above, and the product above that. You see it is just your way upside down.

Cuthbert, an English boy, also learned the new way of multiplying, as it was then called. He also wrote a book, and this is the way he multiplied. (Chart p. 67) He did not know the short way of multiplying by zero.

In Germany it was the custom for boys who intended to go into business, to attend an Arithmetic School. Johann Widman went to such a school and learned to multiply as the other boys did. Later he wrote an arithmetic. He was the first man to put + and — signs in a book. These charts show how multiplication appeared in the early Italian books. (Chart 70 carefully described).

Willis—Oh, these were easy ways and lots of fun.

Division—A long time ago, about the time of Ahmes, the people could not and did not know how to divide. They had a hard time with multiplication, but a still harder time with me. It took many years to learn how to divide, since the people had to know how to multiply first. I want to tell you a little story about a boy who lived at the time of Christopher Columbus. This boy's name was Filippo. He went to a small Italian school. One day the teacher asked Filippo to write two numbers on a small board which he used for his arithmetic. He told him to use the Roman numerals, and then to divide the first number by the second. Of course Filippo could not do this. He said, "I can do it if you will let me use the new figures."

If you had been there you would have thought Filippo's way very queer, for as he finished with a figure he would cross it out. This was called the "scratch method." This is the way it looked, (Chart p. 75) and this is the way he did it. (Division works a problem by the "scratch method" on the blackboard.)

If you had lived when the Pilgrims came to America, you might have learned to divide by the scratch method, because many people used it them. A few years ago I found that they still used the scratch method in Morocco and thought it better than your way. What do you think about it?

Evelyn—Well I think that I like our way best.

Fractions—(Writes on the board when necessary.) Since you have not heard about me, I shall introduce myself as Fractions, and shall tell you how these ancient men learned fractions. Many thousand years ago, when Ahmes learned from the priests in the temple, how to add and subtract, he felt that he had not much more to learn. As he grew older, however, he found that he needed to know something about me. Egypt at that time had no fractions with a numerator greater than one, with the single exception of $\frac{2}{3}$. When the priest and Ahmes spoke of $\frac{3}{4}$, they thought of $\frac{1}{2} + \frac{1}{4}$, and when they spoke of $\frac{7}{8}$, they thought of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$. So it is no wonder that Ahmes had a great deal of trouble in learning fractions, and it is no wonder that he never learned to use them as you do today.

More than a thousand years after Ahmes studied in the temple on the banks of the Nile, there lived in Alexandria at the mouth of the Nile, a boy named Heron. He was interested in machinery, and in measuring heights and distances. He made friends with the scholars in Alexandria who studied the stars. When he made very careful measurements, he found that he needed another kind of fraction. These were fractions that had 60 or 60×60 or $60 \times 60 \times 60$ for their denominator. Just as you write .5 without a denominator, he could write $\frac{23}{60}$ without a denominator. He wrote $\frac{23}{60}$ this way—23', and $\frac{23}{60 \times 60} \times 60$ this way—23'', and so on. Today you still say 23 minutes and 23 seconds, showing that you still use the fractions that Heron used.

There were other ways of writing fractions. How strange it must have been when people wrote fractions this way: L/LIII and VI/VIIII.

Decimals—(Using the blackboard when necessary.) Do you know who I am? I am Decimals. I am first cousin to Miss Fraction, but I am much more important than she is. It was only about three hundred years ago that I came into this great world. Simon Stevin went to a school in Belgium. Like Ahmes and Heron he wrestled with fractions, but while these two boys had despaired of ever learning to conquer them, Simon made up his mind that he would be the victor, and he was. Simon Stevin saw that the common fractions were alright for cases like $\frac{5}{8}$, $\frac{7}{8}$, $\frac{5}{12}$, but they were not good for much when it came to very fine measurements.

The world was much interested in science, especially astronomy, and so needed better fractions. Simon Stevin wrote the first book on decimals. This is the way he wrote 12.754; 1 2⓪ 7① 5② 4③ You see that he did not use the decimal point that you use.

Decimals, like fashions, change, and so in different countries people write them in different ways. In England they write 3·4, with the point halfway up; in France and Germany, 3,14, using the comma; and in your country 3.14. And strange to say, you all call it three and fourteen hundredths.

Evelyn—Why, I never knew the story of mathematics was so interesting.

Jane—Well, we have had a real mathematics lesson.

Willis—Yes, but this was interesting!

Grandfather—You will find everything interesting, if you just look at it properly. And now we must say good bye to these interesting people and their wonderful land.—Curtain.

THIRD ACT.

Children working at table, left side of the stage. Grandfather and mother seated near center. All work done on board and explained when necessary.

Grandfather—So this is to be our last night.

Evelyn—Last but one.

Grandfather—Are you sure?

Children—Sure!

Grandfather—What shall it be tonight, square root or cube root?

Willis—Square root doesn't sound interesting.

Grandfather—Logarithms?

Jane—Never heard of them. Tell us something interesting.

Grandfather—Well, as we have a good log fire here, let us imagine it a cold night, and this place at the edge of a dense forest; the house being built of logs, and having a huge fire-place, with many comfortable chairs arranged around it. Over the heavy door, someone has written in the years gone by, these words, "Let no one without imagination enter here." So the door is closed to you and me and to all the world if we have not imagination. For us the log fire is cold and the candles do not burn, if we belong to that humdrum class that never has any day dreams. You then without any imagination, who belong to those unfortunates, who believe only what they hear, stop on the threshold. For you the log fire has no attraction, and the empty chairs no occupants. (As Grandfather calls up these guests, they appear silently from each side, carrying stools and arrange themselves in a semi-circle facing the audience. The children, surprised, seat themselves on the floor, in the center of the stage, facing the strangers.) From out of the great unknown of time and space, they come, one by one to the house of logs, our friends of long ago. Chang from the Land of the Yellow Dragon; Anam from the Tigris; Menes and Ahmes from the banks of the Nile; Hippias from Greece; Titus from the Seven Hills of Rome; Daniel from the Mount of Olives; Gupta and Gerbert, and most of the others of whom we have read. Strangers to each other, strangers to the present civilization, but drawn together by their interest in the puzzles of the world of number. Now if you will watch and listen, perhaps our friends will tell you something interesting.

Chang—In my land there is an ancient book, perhaps the oldest in the world. The name of this book is "Yii King". It was written thousands of years ago, and records one of the earliest of all curious things connected with number. It says that once upon a time, there came out of the Yellow River, a large turtle, and on its back were strange marks, which puzzled everybody who saw them. These I have copied on paper for you to see. (Shows chart p. 97).

Gupta—Oh, I know what that is. It is a magic square and the dots are numbers. The columns add to fifteen and

4	9	2
3	5	7
8	1	6

so do the rows. Now watch the diagonals. It is the world's greatest number mystery, and is used as a charm all through the East and in the Middle ages, long after I died, it was used in many parts of Europe to drive away disease and bring good fortune.

Grandfather—Yes, Gupta is right, for in the Magic Square we have one of the oldest and most interesting curiosities to be found anywhere in the world.

Menes—When I was living in Egypt in the early days, we did not have many number puzzles. But since then I have watched the world and have seen many strange things happen. One of the most curious relates to the numbers now used in Europe and America and all places under their influence. If you take any number, say 3467 and reverse it 7643 the difference will always be divisible by 9. Try this and see if it is not true.

Chang—I have it.
$$\begin{array}{r} 3827 \\ 2783 \\ 9)1044 \\ 116 \end{array}$$

Daniel—You did that wrong. It should be 3872.

Menes—That does not make any difference. You may mix the figures any way you wish, and the rule still holds. After you have studied a little algebra, you will see how this is done, for algebra is like a great electric light; it reveals all the secrets of arithmetic.

Heron—One of the most curious things that I have noticed in looking at the world for the past two thousand years, is seen in a simple problem in addition. If you write any three numbers, say numbers of four figures each, I will at once write three numbers underneath, and before doing so will tell you the sum of all six numbers. It does not make any difference what numbers you take, nor does it make any difference whether they are all alike or all different.

Adrien—I will write these numbers.

$$\begin{array}{r} \left.\begin{array}{r} 7632 \\ 5847 \\ 6195 \end{array}\right\} \text{Written by Adrien.} \\ \left.\begin{array}{r} 2367 \\ 4152 \\ 3804 \end{array}\right\} \text{Written by Heron.} \\ \hline 29997 \end{array}$$

Heron—The sum of all six numbers will be 29997.

Evelyn—I am going to try that and see if it is right. (Adds the columns aloud.)

Heron—It would be fun for all of you to try it and see how it is done.

Gerbert—A strange thing was shown to me when I went as a boy to Barcelona. Any one of you may write any number of as many places as you please, and I will at once write a figure at the end of your number and make it exactly divisible by 11.

Jakob—I will write that number, (He wrote 74289).

Gerbert—(Writes 6 at the end of the number, and divides it by 11.) Now would you like me to show you how I did it?

All—Yes. Show us.

Gerbert—9 and 2 is 11, and 7 is 18. 8 and 4 is 12, and you must add 6 to make it 18. I have now given you good measure, for I have made this number exactly divisible by 22, 33, and 99.

Michael Stiffel—When I was a boy we had a couplet that ran like this, "Ten fingers have I on each hand, Five and twenty on hands and feet." You know we used to say five and twenty, for twenty-five.

Chang—But you have not five and twenty fingers on your hands and feet.

Cuthbert—You must count your toes as fingers.

Hippias—Even then we have only twenty.

Michael Stiffel—It is perfectly easy if you only know how to say it. Shall I tell you?

All—Yes, do tell us.

Michael—Ten fingers have I; on each hand five; and twenty on hands and feet.

Evelyn—Well that was a good one on us.

Jakob—I believe that one of the oddest things that I ever saw in numbers, is a set of products which I shall show you.

$$\begin{aligned} 3 \times 37 &= 111 \\ 6 \times 37 &= 222 \\ 9 \times 37 &= 333 \\ 12 \times 37 &= 444 \end{aligned}$$

Now can anyone tell me what 15×37 is?

Willis—I guess it will be 555.

Jakob—You are right. Another strange thing is that $1 + 1 + 1$ is 3; $2 + 2 + 2$ is 6; $3 + 3 + 3$ is 9, and so on.

Anam—While you are talking about such curious things, I will show you something really interesting.

$$\begin{aligned} 1 \times 8 + 1 &= 9 \\ 12 \times 8 + 2 &= 98 \\ 123 \times 8 + 3 &= 987 \\ 1234 \times 8 + 4 &= 9876 \\ 12345 \times 8 + 5 &= 98765 \end{aligned}$$

and so on. This is how it looks when it is finished. (Shows chart p. 106). Now when we go home, try it and see if it is correct.

Menes—Yes that is very interesting, but I will show you something quite as curious.

$$1 \times 9 + 2 = 11$$
$$12 \times 9 + 3 = 111$$
$$123 \times 9 + 4 = 1111$$
$$1234 \times 9 + 5 = 11111$$

If you continue this until you have used all the numerals, your products will still contain only ones.

Ahmes—It seems that these figures which were invented many hundreds of years after I died are very queer indeed. You could never have done that with the figures that I learned four thousand years ago.

Cuthbert—Yes, and every one knows that no one else could do anything with them, they were so bad.

Ahmes—Well you need not be so proud of your numerals. Try writing twelve thousand, twelve hundred, twelve.

Cuthbert—Why, that's easy (Writes 121212.)

Ahmes—Now see what you have written.

Cuthbert—Oh that's 121,212. Well I guess it can't be done.

Ahmes—In the earlier days we would have written it something like this: 12M 12C 12. Of course we would have used other marks for the numbers, and signs for thousand and hundred.

Titus—When I was a boy, I was given a puzzle problem, which I will have to change a little so as to give it to you in English. 6—9 9—10 40—50 Can any of you do this so as to have six for a remainder?

Children—It can't be done.

Titus—Yes it can.

SIX	IX	XL	
IX	X	L	
S	I	X	spells six.

Adrien—Here is a good one. Show me how to write one hundred using the nine digits and the signs of arithmetic.

Hippias—Impossible.

Adrien—I will show you. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \times 9$ equals 100.

Daniel—It does not equal 100. (Adds all the numbers through 8 and then multiplies the result by 9 and gets 324.) And 324 is not 100.

Adrien—You did that wrong. 1, 3, 6, 10, 15, 21, 28; $8 \times 9 = 72$; 72 plus 28 is 100. You do not know that you must multiply first.

Daniel—Oh, I forgot.

Gupta—Who can write four with four nines.

Hippias—That is too easy. Why don't you ask who can write twenty using four nines. That is a problem worth thinking about. I will show you all of them.

$$\frac{9}{9} + \frac{9}{9} = 2 \qquad \frac{99}{9} + 9 = 20$$

Can anyone write 100 without using any zero's?

Cuthbert—$99 + \frac{9}{9} = 100$. Now I have shown you how to write an even number using only odd digits.

Ahmes—Now before we go, I am going to tell you a good one. Two fathers and two sons divided three apples equally amongst themselves, so that each received a whole apple. How was this possible?

Hippias—They had to cut the apples.

Ahmes—No they did not. Shall I tell you?

All—Yes tell us.

Ahmes—One was the grandfather, one the father, and the other the son. Were there not two fathers, and two sons? (The clock strikes twelve and the visitors disappear. The children return to their work and look up startled to find themselves alone.)

Evelyn—Why, where are all our friends?

Grandfather—They have gone. They came because you had good imaginations. Now I only hope you will remember all the interesting things they have shown you. The evening is late and we must say good night.

END.